ELECTROMAGNETIC INDUCTION

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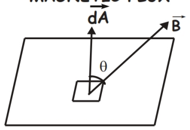
MAGNETIC FLUX $\propto$ no of lines of force



$d\Phi_B = \vec{B}.d\vec{A} = BdA\cos\theta$

$\Phi_B = \int \vec{B}.d\vec{A} = \int BdA\cos\theta$

for uniform field, $\vec{B}$ $\Phi_B = \vec{B}.\vec{A}$

$\Phi_B = BA\cos\theta$

- Scalar quantity ⟶ Unit ⟶ Weber(Wb)
- $[\Phi_B] = ML^2T^{-2}A^{-1}$

FARADAY'S LAW

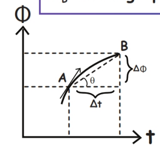
1) Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit
2) The induced EMF is given by rate of change of magnetic flux linked with the circuit

$\mathcal{E}_{Ind} = -\frac{d\Phi_B}{dt}$

Negative sign indicates that induced emf opposes the cause of flux change

$\mathcal{E}_{Ind} = -\frac{Nd\Phi_B}{dt}$

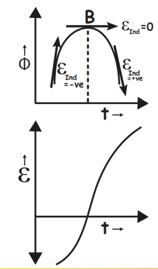
References from Φ_B V/S t graph



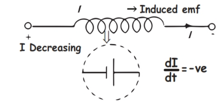
Slope of chord AB in Φ-t Graph $= \mathcal{E}_{Ind} = \frac{\Delta\Phi}{\Delta t}$

Slope of the tangent in the Φ-t Graph $= |\mathcal{E}_{Ind}| = \frac{d\Phi}{dt}$

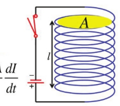
CHANGING Φ-t GRAPH INTO OTHER GRAPH



CASE 2



self inductance of a long solenoid



$B = \frac{\mu_0 NI}{\ell}$ $\quad \Phi_{B_1} = \frac{\mu_0 NIA}{\ell}$

$E = -\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\Phi_{B_1} = -\frac{d}{dt}\left(\frac{\mu_0 N^2 IA}{\ell}\right) = -\frac{\mu_0 N^2 A}{\ell}\frac{dI}{dt}$

$E = -L\frac{dI}{dt}$ $\quad L = \mu_0 N^2 A/\ell$

L=Coefficient of self inductance $L = \mu_0 N^2 A/l$

$L = \mu_0 n^2 Al$

Unit of inductance: Henry [H] $= [ML^2T^{-2}A^{-2}]$

MUTUAL INDUCTANCE

Change in current in one coil causes change in flux in another coil and vice versa.

Let there be two coils A and B, having currents I_1 and I_2.

$\phi_B \propto I_1$ $\qquad \phi_A \propto I_2$

$\Rightarrow \phi_B = MI_1$ $\qquad \Rightarrow \phi_A = MI_2$

M is called as mutual inductance of the coils.

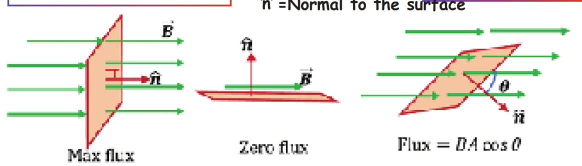
EMF induced,

$\mathcal{E}_B = -\frac{d\phi_B}{dt} \Rightarrow \mathcal{E}_B = -M\frac{dI_1}{dt}$

& $\mathcal{E}_A = -\frac{d\phi_A}{dt} \Rightarrow \mathcal{E}_A = -M\frac{dI_2}{dt}$

In uniform $\vec{B}$

$\hat{n}$ = Normal to the surface



$\Phi_B = BA$ (maximum)

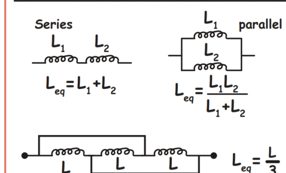
ENERGY STORED IN INDUCTOR

$U_B = \frac{1}{2}LI^2$

MAGNETIC ENERGY STORED PER UNIT VOLUME ENERGY DENSITY

$u = \frac{U_B}{V} = \frac{U_B}{Al} = \frac{\mu_0 n^2 I^2}{2} = \frac{B^2}{2\mu_0}$

SERIES & PARALLEL COMBINATION OF INDUCTORS



Series: $L_{eq} = L_1 + L_2$

parallel: $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

$L_{eq} = \frac{L}{3}$

LENZ'S LAW & CONSERVATION OF ENERGY

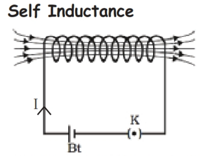
The direction of any induced magnetic effect is such as to oppose the change that produces it

INDUCTANCE

- Scalar quantity
- Unit of inductance (H)
- Dimension : $ML^2T^{-2}A^{-2}$

INDUCTANCE → Self Inductance, Mutual Inductance

Self Inductance



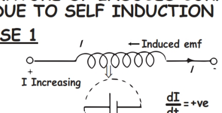
Current I in the coil changes due to external source
↓
Causes change in magnetic field inside the coil
↓
Results in change in magnetic flux inside the coil
↓
EMF is induced which opposes the changing magnetic flux
↓
Creates an induced current which is opposing in nature

$E = -L\frac{dI}{dt}$

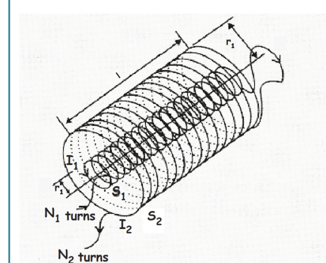
where, L = Self Inductance
I = Current in the coil

NATURE OF INDUCED CURRENT DUE TO SELF INDUCTION

CASE 1



MUTUAL INDUCTANCE OF TWO CO-AXIAL SOLENOIDS



I_2 = Current through outer coil

$B = \mu_0 n_2 I_2$

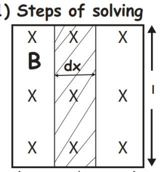
$\Phi_{12} = \mu_0 n_2 I_2 \times \pi r_1^2 \, n_1 l$

$M = M_{12} = \mu_0 n_1 n_2 \times \pi r_1^2 \, l$

$= \frac{\mu_0 N_1 N_2}{l} \pi r_1^2$

FLUX IN NON-UNIFORM FIELD

1) Steps of solving



1. Take a small strip 'dx'
2. flux $d\Phi = BdA$ $\quad [dA = ldx]$
3. Total flux $\Phi = \int d\Phi = \int_0^b BdA = \int_0^b Bldx$

EMF

Average value

$\mathcal{E}_{Ind} = -\frac{\Delta\Phi}{\Delta t}$

$[\Delta\Phi = \Phi_2 - \Phi_1]$

Instantaneous value

$\mathcal{E}_{Ind} = -\frac{d\Phi}{dt}$

Induced current

$I_{ind} = \frac{\mathcal{E}_{Ind}}{R} = -\frac{1}{R}\frac{\Delta\Phi}{\Delta t}$ $\qquad I_{ind} = \frac{\mathcal{E}_{Ind}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}$

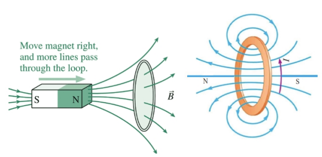
Charge flown due to induced current

$q_{flown} = \frac{\Delta\Phi}{R}$

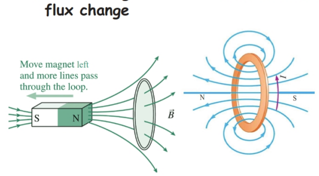
Charge flown due to induced current depends only on change in flux

DIRECTION OF INDUCED CURRENT

1. If flux is decreasing, the magnetic field due to induced current will be along the existing magnetic field
2. If flux is increasing, the magnetic field due to induced current will be opposite to existing magnetic field

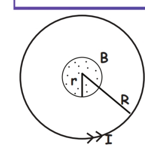


Field causing flux change $\qquad$ Induced Field



Field causing flux change $\qquad$ Induced Field

MUTUAL INDUCTANCE OF SOME STANDARD CASES



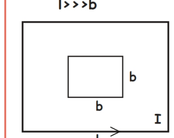
R>>>r

$\Phi = BA$ $\quad B = \frac{\mu_0 I}{2R}$

$\Phi = \frac{\mu_0 I \times \pi r^2}{2R} = MI$

$M = \frac{\mu_0 \times \pi r^2}{2R}$

l>>>b



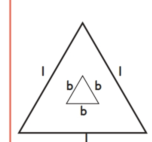
$\Phi = BA$

$B = \frac{\mu_0 2\sqrt{2}}{\pi l} I$

$\Phi = \frac{\mu_0 2\sqrt{2}}{\pi \ell} I \times b^2$

$M = \frac{\mu_0 2\sqrt{2}}{\pi \ell} \times b^2$

l>>>b



$\Phi = BA$

$B = \frac{9}{2}\frac{\mu_0 I}{\pi l}$

$\Phi = BA = \frac{9}{2}\frac{\mu_0 I}{\pi l} \times \frac{\sqrt{3}}{4} b^2$

$M = \frac{9}{2}\frac{\mu_0}{\pi l} \times \frac{\sqrt{3}}{4} b^2$

Relation between mutual inductance & self inductance

$M = K\sqrt{L_1L_2}$ $0 \le K \le 1$

$K \rightarrow$ coefficient of coupling

If K=1, Perfect flux linkage

$M = \sqrt{L_1L_2}$

Otherwise $\rightarrow$ imperfect linkage

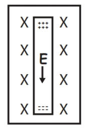
K<l

If K=0

no linkage

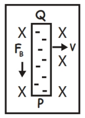
$\Rightarrow M=0$

DYNAMIC MOTIONAL EMF DUE TO TRANSLATORY MOTION



- Charges accumulate at the ends of the conductor due to its movement in external magnetic field
- This separation of charges at the ends of the conductor causes a voltage difference

At steady state



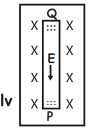
$F_B = F_e$

$evB = eE$

$E = vB$

$\frac{V_{PQ}}{l} = vB$

V_{PQ} or $\mathcal{E} = Blv$



Direction to find the positive terminal

By using right hand rule,

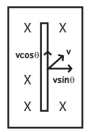
Thumb - velocity

Fingers - Magnetic field

Palm - Positive terminal of the rod

Modification

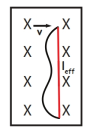
i) Velocity is not perpendicular



$\mathcal{E} = Blv\sin\theta$

ii) Conductor of arbitrary shape

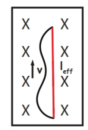
$\vec{v}$ Perpendicular to effective length



l_{eff} = effective length

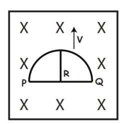
$\mathcal{E}_{ind} = Bl_{eff}v$

$\vec{v}$ Parallel to effective length



$\mathcal{E}_{ind} = 0$

Semi circular loop in a magnetic field



$l_{eff} = 2R$

$\mathcal{E} = Bl_{eff}v$

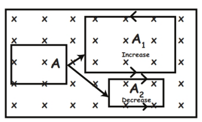
$\mathcal{E} = Bx2Rv$

from right hand rule,

P is at higher potential

Q is at lower potential

change of area in magnetic field region



$\phi = BA$

$\Delta\phi = B\Delta A$

$\mathcal{E}_{Ind} = \frac{\Delta\phi}{\Delta t} = \frac{B\Delta A}{\Delta t}$

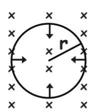
$A \rightarrow A_1 \rightarrow$ Area increases, current will be anticlockwise

$A \rightarrow A_2 \rightarrow$ Area decreases, current will be clockwise

Shrinking Loop

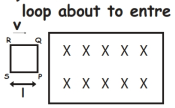
Loop shrinking at rate $\frac{dr}{dt}$

$\mathcal{E}_{Ind} = Bx2\pi rx\frac{dr}{dt}$



MOTIONAL EMF : FARADAY'S LAW

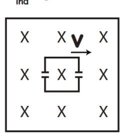
i) At t=0 $\rightarrow$ loop about to entre



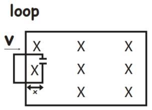
iii) Loop has fully entered

$\mathcal{E}_{Ind} = 0$

$I_{ind} = 0$



ii) At time t=t, 'x' length inside loop



$A = lx$

$B_{ind} \rightarrow \odot$

$I_{ind} \rightarrow$ anticlockwise

$\phi = BA = Blx$

x=vt

$\phi = Blvt$

$\mathcal{E}_{Ind} = Blv$

$I_{ind} = \frac{\mathcal{E}_{Ind}}{R} = \frac{Blv}{R}$

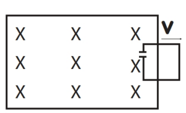
iv) Loop exiting. Field is decreasing

$B_{ind} \otimes$

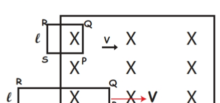
I_{ind} clockwise

$\mathcal{E}_{Ind} = Blv$

$I_{ind} = \frac{Blv}{R}$



MOTION OF A SQUARE, RECTANGLE, CIRCLE & ELLIPSE IN UNIFORM MAGNETIC FIELD



For square,

$PQ \rightarrow \mathcal{E} = Blv =$ Constant

RQ & SP $\rightarrow l_{eff}$ Parallel to velocity

RS $\rightarrow \mathcal{E} = 0$ (Outside B)

Q - Higher potential

P - Lower potential

Current - Anticlockwise direction

For Rectangle

$PQ \rightarrow \mathcal{E} = Blv =$ Constant

RQ & SP $\rightarrow l_{eff}$ Parallel to velocity

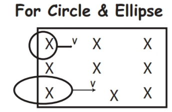
RS $\rightarrow \mathcal{E} = 0$ (Outside B)

Q - Higher potential

P - Lower potential

Current - Anticlockwise direction

For Circle & Ellipse



Effective length is constantly varying

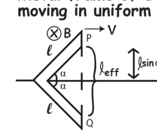
Induced EMF is varying. When loop is entering, EMF first increases reaches to a maximum and then decreases

induced current in anticlockwise direction

Instantaneous Induced EMF = $B(l_{eff})_{inst}\,v$

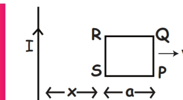
TRANSLATORY MOTION OF METALLIC FRAME IN UNIFORM / NON UNIFORM MAGNETIC FIELD

Metal frame of different shapes moving in uniform magnetic field



$V_P - V_Q = 2Bl\sin\alpha v$

MOVING METAL FRAME IN NON UNIFORM MAGNETIC FIELD

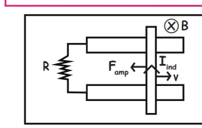


$B_{at\,RS} = \frac{\mu_0 I}{2\pi x}$, $\mathcal{E}_{RS} = \frac{\mu_0 I}{2\pi x}av$

$B_{at\,QP} = \frac{\mu_0 I}{2\pi(x+a)}$, $\mathcal{E}_{QP} = \frac{\mu_0 I}{2\pi(x+a)}av$

$\mathcal{E}_{net} = \mathcal{E}_1 - \mathcal{E}_2 = \frac{\mu_0 Ia^2v}{2\pi x(x+a)}$

INDUCED CURRENT AND AMPERIAN FORCE



- Amperian force

$F_{amp} = \frac{B^2l^2v}{R} \rightarrow$ Opposes motion

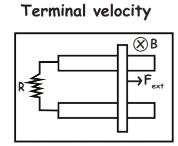
- Work done by amperian force

$W = \Delta KE = 0 - \frac{1}{2}mv_0^2$ [Initial vel.= v_0, Final vel.= 0]

- Power developed in circuit

$P = I^2R$ $P = \frac{B^2l^2v^2}{R}$

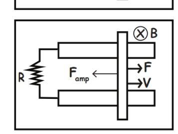
Terminal velocity



t=0

slider at rest

F_{ext} starts acting on rod



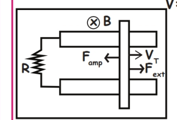
t=t

$F_{amp} = \frac{B^2l^2v}{R}$

when is terminal velocity achieved

a=0

$V = V_T$ = constant

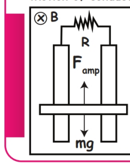


$F = \frac{B^2l^2V_T}{R}$

At t=∞

$V_T = \frac{FR}{B^2l^2}$

Motion of conductor in a vertical plane



At $V = V_T$

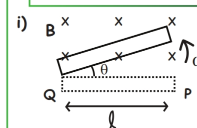
$F_{amp} = mg$

$mg = \frac{B^2l^2V_T}{R}$

$V_T = \frac{mgR}{B^2l^2}$

MOTIONAL EMF DUE TO ROTATIONAL MOTION

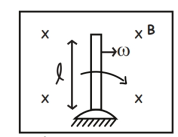
i)



$\mathcal{E} = \frac{1}{2}Bl^2\omega$

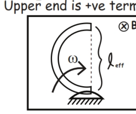
End Q is positive terminal

ii)



$\mathcal{E} = \frac{1}{2}B\ell^2\omega$

Upper end is +ve terminal



$\mathcal{E} = \frac{1}{2}B(4R^2)\omega$

Lower end is +ve

INSTANTANEOUS VALUE OF INDUCED EMF

ϕ = NBA cosθ

= NBA cosωt

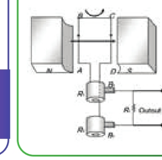
$\mathcal{E}_{Ind}$ = NBAω sinωt

$\mathcal{E}_0$ = NBAω

when t=0

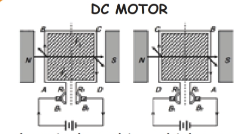
$\mathcal{E}_{Ind} = \mathcal{E}_0\sin\omega t$

AC GENERATOR



$\mathcal{E}_{Ind} = \mathcal{E}_0\sin\omega t$

DC MOTOR



electrical machine which converts electrical energy into mechanical energy

EQUATION FOR BACK EMF

$I = \frac{E - NBA\omega\sin\omega t}{R}$

MECHANICAL POWER & EFFICIENCY OF DC MOTOR

$\eta = \frac{\text{back emf}}{\text{supply voltage}} = \frac{P_{out}}{P_{in}} = \frac{e}{E}$

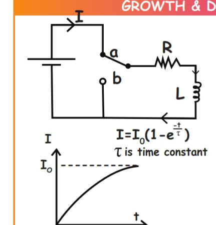
L-R CIRCUIT

Steady state $\rightarrow$ inductor $\rightarrow$ zero resistance

For current growth in a circuit

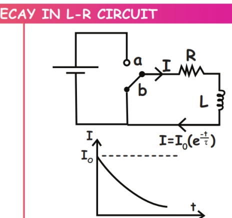
- at t=0 Inductor offers infinite resistance
- at t=∞ Inductor offers zero resistance

GROWTH & DECAY IN L-R CIRCUIT



$I = I_0(1 - e^{-\frac{t}{\tau}})$

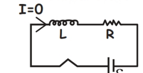
τ is time constant



$I = I_0(e^{-\frac{t}{\tau}})$

INITIAL & FINAL STATE OF L-R CIRCUIT

Initial state

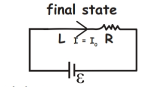


at t=0

Initial state - inductor open

$\mathcal{E} = V_L$

final state

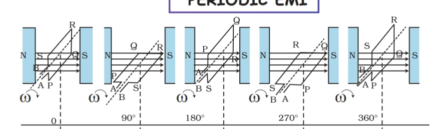


at t=∞

final state - replace inductor with a wire

$I_0 = \frac{\mathcal{E}}{R}$

PERIODIC EMI



ϕ_1=NBAcos0 =NBA ϕ_2=NBAcos90 =0 ϕ_3=NBAcos180 =-NBA ϕ_4=NBAcos270 =0 ϕ_5=NBAcos360 =NBA

Average induced emf

- when coil is rotated from $\theta = 0^0$ to 90^0 $\mathcal{E}_{ind} = \frac{2NBA\omega}{\pi}$
- when coil is rotated from $\theta = 90^0$ to 180^0 $\mathcal{E}_{ind} = \frac{-2NBA\omega}{\pi}$